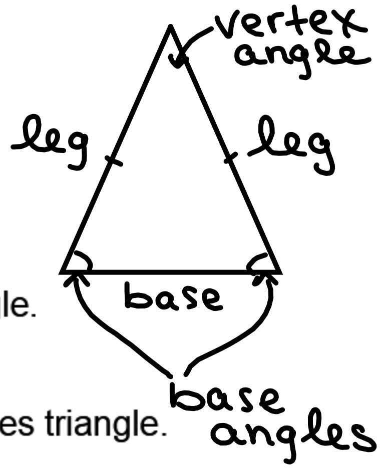
Isosceles and Equilateral Triangles

After today, we should be able to use and apply theorems and properties of isosceles and equilateral triangles.



Vocabulary

1. Legs of an isosceles triangle
   The congruent sides of an isosceles triangle.

2. Vertex angle
   The angle formed by the legs of isosceles triangle.

3. Base
   The side opposite the vertex angle of an isosceles triangle.

4. Base angles
   The two angles that have the same base as a side.

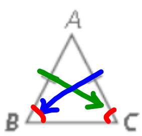
Theorems—Isosceles Triangle

Isosceles Triangle Theorem

Hyp. conc.

If two sides of a triangle are congruent, then the angles opposite the sides are congruent.

HYPOTHESIS



The legs are congruent.
$\overline{AB} \cong \overline{AC}$

CONCLUSION

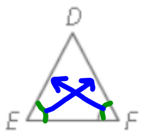
The base angles are congruent.
$\angle B \cong \angle C$

Converse of Isosceles Triangle Theorem → swap the hypothesis/conclusion.

Hyp. conc.

If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

HYPOTHESIS



The base angles are congruent.
$\angle E \cong \angle F$

CONCLUSION

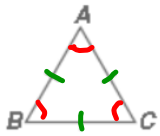
The legs are congruent.
$\overline{DE} \cong \overline{DF}$

Corollary Equilateral Triangle

Hyp. conc.

If a triangle is equilateral, then it is also equiangular.
(equilateral $\Delta \rightarrow$ equiangular Δ)

HYPOTHESIS



All three sides are congruent.
$\overline{AB} \cong \overline{BC} \cong \overline{AC}$

CONCLUSION

All three angles are congruent.
$\angle A \cong \angle B \cong \angle C$

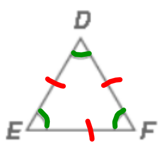
Corollary Equiangular Triangle

Hyp. conc.

If a triangle is equiangular, then it is also equilateral.
(equiangular $\Delta \rightarrow$ equilateral Δ)

This is the converse of the previous corollary.

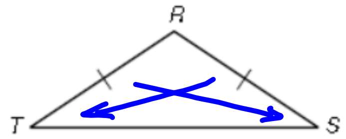
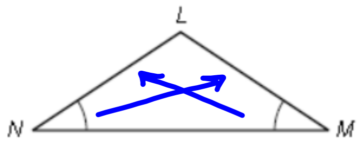
HYPOTHESIS



All three angles are congruent.
$\angle D \cong \angle E \cong \angle F$

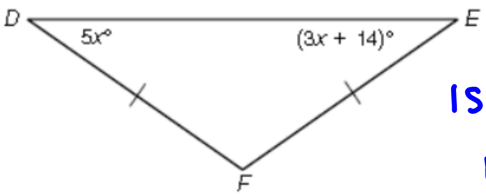
CONCLUSION

All three sides are congruent.
$\overline{DE} \cong \overline{DF} \cong \overline{EF}$

Theorem	Examples
Isosceles Triangle Theorem If two sides of a triangle are congruent, then the angles opposite the sides are congruent.	 If $\overline{RT} \cong \overline{RS}$, then $\angle T \cong \angle S$.
Converse of Isosceles Triangle Theorem If two angles of a triangle are congruent, then the sides opposite those angles are congruent.	 If $\angle N \cong \angle M$, then $\overline{LN} \cong \overline{LM}$.

You can use these theorems to find angle measures in isosceles triangles.

$\triangle DEF$ is isosceles because $\overline{DF} \cong \overline{EF}$.



Find $m\angle E$ in $\triangle DEF$.

Isosceles $\triangle$ Thm $\rightarrow \angle D \cong \angle E$.

$m\angle D = m\angle E$

$5x = 3x + 14$

$-3x \quad -3x$

$\frac{2x}{2} = \frac{14}{2}$

$x = 7$

$m\angle E = 3(7) + 14$

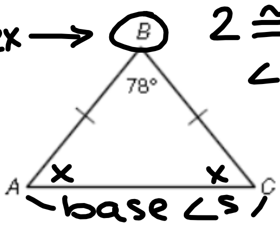
$21 + 14$

35

$m\angle D = 5(7) = 35$ ✓

Find each angle measure.

vertex ∠ →

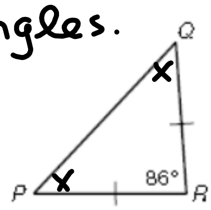


2 ≅ sides → 2 ≅ angles.

$\angle A \cong \angle C$

1. $m\angle C =$ 51°

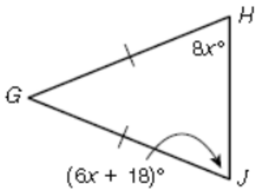
$$78 + 2x = 180$$
$$-78 \quad -78$$
$$\frac{2x}{2} = \frac{102}{2}$$
$x = 51$



$\angle P \cong \angle Q$

2. $m\angle Q =$ 47°

$$86 + 2x = 180$$
$$-86 \quad -86$$
$$\frac{2x}{2} = \frac{94}{2}$$
$x = 47$



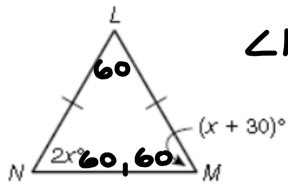
$\angle H \cong \angle J$

3. $m\angle H =$ 72°

$$m\angle H = m\angle J$$
$$8x = 6x + 18$$
$$-6x \quad -6x$$
$$\frac{2x}{2} = \frac{18}{2}$$
$x = 9$

$m\angle H = 8(9)$
72

$m\angle J = 6(9) + 18$
54 + 18
72 ✓



$\angle N \cong \angle M$

4. $m\angle M =$ 60°

$$m\angle M = m\angle N$$
$$x + 30 = 2x$$
$$-x \quad -x$$
$$30 = x$$

$m\angle M = 30 + 30 = 60$

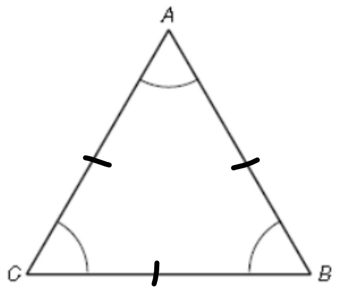
$m\angle N = 2(30) = 60$ ✓

Equilateral Triangle Corollary
If a triangle is equilateral, then it is equiangular.

(equilateral △ → equiangular △)

(converse)
Equiangular Triangle Corollary
If a triangle is equiangular, then it is equilateral.

(equiangular △ → equilateral △)



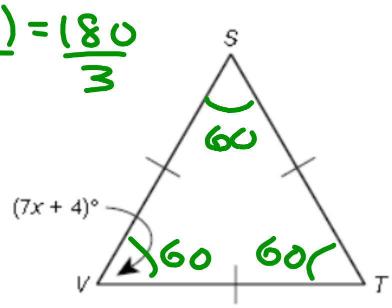
If $\angle A \cong \angle B \cong \angle C$, then $\overline{AB} \cong \overline{BC} \cong \overline{CA}$.

You can use these theorems to find values in equilateral triangles.

Find *x* in △*STV*.

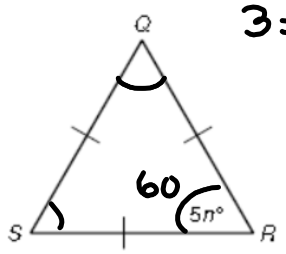
$$\frac{3(7x+4)}{3} = \frac{180}{3}$$

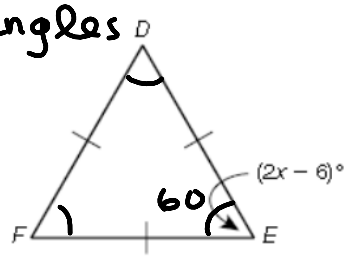
$$7x + 4 = 60$$
$$-4 \quad -4$$
$$\frac{7x}{7} = \frac{56}{7}$$
$$x = 8$$



Find each value.

3 ≅ sides → 3 ≅ angles





5. $n =$ 12

$$\frac{5n}{5} = \frac{60}{5}$$

$n = 12$

6. $x =$ 33

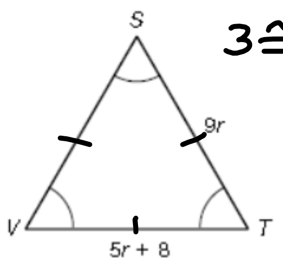
$2x - 6 = 60$

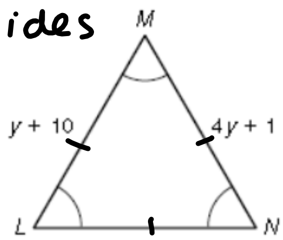
$+6 \mid +6$

$$\frac{2x}{2} = \frac{66}{2}$$

$x = 33$

3 ≅ angles → 3 ≅ sides





7. $VT =$ 18

$VT = ST$

$5r + 8 = 9r$

$-5r \mid -5r$

$$\frac{8}{4} = \frac{4r}{4}$$

$r = 2$

$VT = 5(2) + 8$

$10 + 8$

18

$ST = 9(2) = 18$ ✓

8. $MN =$ 13

$LM = MN$

$y + 10 = 4y + 1$

$-y - 1 \mid -y - 1$

$$\frac{9}{3} = \frac{3y}{3}$$

$y = 3$

$MN = 4(3) + 1 = 13$

$LM = 3 + 10 = 13$ ✓